A Parametric Study of Gas-Particle Flows in Conical Nozzles

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A parametric study of two-dimensional gas-particle flow effects in conical nozzles is presented. Values of particle throat velocity lag, total mass flow rate, and specific impulses are presented for various cone angles, expansion ratios, throat radii, particle sizes, and particle flow rate ratios for fixed values of chamber pressure and temperature, vacuum ambient pressure, and a gas composition that varies with temperature but is independent of the particle flow rate ratio.

Nomenclature

C_p = specific heat at constant pressure of gas
D = particle diameter, μ
I_{sp} = specific impulse
K_t = particle velocity lag at nozzle throat
p_0 = ambient pressure
P_c = chamber pressure
Pr = Prandtl number
R = gas constant
T_c = chamber temperature
T_g = gas temperature
$\dot{w}$ = total mass flow rate
$\dot{w}_g$ = gas mass flow rate
$\dot{w}_p$ = particle mass flow rate
y_t = throat radius, in.
α = nozzle exit semiangle
β = nozzle entrance semiangle
γ = specific heat ratio of gas
δ = flow rate ratio ($\dot{w}_p/\dot{w}_g$)
ϵ = nozzle exit area ratio
μ = viscosity of gas
ρ_d = nozzle throat downstream radius of curvature, in.
ρ_u = nozzle throat upstream radius of curvature, in.

Introduction

THE importance of utilizing powdered metal additives in propellant formulations to increase the energy release of the propellant and to suppress combustion instability is well recognized. However, significant performance losses relative to isentropic thermochemical predictions occur as a result of the nonequilibrium flow effects associated with particle drag and heat transfer. The analyses presented in Refs. 1 and 2 appear to describe the flow of gas-particle mixtures adequately in rocket nozzles.

Because of the scale effects associated with gas-particle flow, general solutions for similar nozzles cannot be obtained. Thus, the present investigation is concerned with a parametric study of the effects of particle nonequilibrium flow on rocket motor specific impulses for typical operating conditions in conical nozzles. The results were obtained by applying the numerical method of characteristics to the axisymmetric flow field in the nozzle and by accounting for particle drag, heat transfer, slip flow, and a typical variation in gas properties. The effects of cone angle, expansion ratio, throat radius, particle size, and particle flow rate ratio were investigated. The results presented in the figures were obtained with an IBM 7090 computer program based on the method of Ref. 1 for axisymmetric flow.

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Analysis

The present analysis is based on the usual assumptions for axisymmetric gas-particle flows[1] and considers, in addition, the effects of slip flow, variable gas properties, and particle drag and heat-transfer phenomena, which are not restricted to Stokes flow. The flow in the subsonic portion of the nozzle is assumed to be one-dimensional, and the analysis presented by Kliegel[3] is applicable. By employing that analysis, the particle velocity lags can be evaluated at the nozzle throat. As shown by Kliegel,[3] the throat region of a DeLaval nozzle can be considered as a constant fractional-lag flow region. Subject to that assumption, a modified perfect gas flow is assumed to exist in the throat region, and Sauer's analysis[4] is applied to obtain an initial value line for the modified perfect gas flow at the nozzle throat where the actual gas flow is subsonic. The numerical method of characteristics is then utilized to obtain the gas properties in the transonic flow regime downstream of the initial value line. The particle trajectories are then traced through the transonic throat region to obtain particle properties along an initial value line at which the actual gas flow is supersonic. The numerical method of characteristics is then applied to the axisymmetric flow of the actual gas-particle mixture to obtain the flow field in the supersonic portion of the nozzle. Nozzle mass flow is obtained by integration of mass flow across the initial value line, and nozzle thrust is obtained by integration of pressure forces along the wall and momentum and pressure forces across the initial value line. As pointed out by Hoglund,[5] the main limitation in the application of this theory is the accuracy with which the required gas and particle thermodynamic and transport properties can be evaluated.

Numerical Results

Any parametric study of gas-particle effects in axisymmetric nozzles must be limited in scope because of the scale effects associated with the particles. The data considered in this investigation are typical values associated with current high-energy aluminized propellants. The particles in all cases are

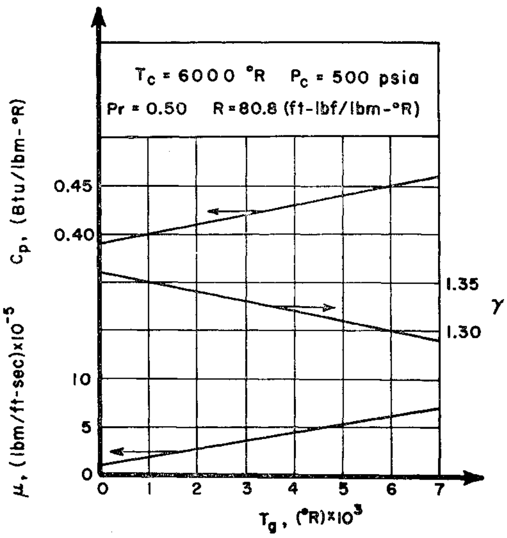

Fig. 1 Gas thermodynamic and transport properties.

aluminum oxide and are considered to be of one average size in order to study the effect of particle size directly. The thermodynamic properties of the particles were taken from the JANAF Thermochemical Data Tables. Drag coefficients were taken from Schlichting,[6] heat-transfer coefficients were based on the work of Drake,[7] and rarefaction corrections were taken from Schaaf and Chambre.[8]

The gas thermodynamic and transport properties utilized throughout the investigation are shown in Fig. 1. These data are assumed to be independent of the particle flow rate ratio, an assumption certainly not true for any actual propellant formulation. Variations in the particle flow rate ratio will affect specific impulse most directly through changes in combustion temperature and gas constant, whereas the accompanying variations in the gas thermodynamic and transport properties should exert only a secondary influence. The combustion temperature is 6000°R, and the gas constant is 80.8 ft-lbf/lbm-°R in all of the figures presented herein. However, combustion temperatures in the range 5500°–6500°R and gas constants in the range 65–85 ft-lbf/lbm-°R were also investigated. It was found that the results presented in Figs. 2–7 could be extended to include the preceding range of combustion temperatures and gas constants, predicting specific impulse within ± 1%, by applying the correction factor $[(RT_c)/(80.8*6000)]^{1/2}$ based on the square root of the ratio of the products of gas constant and combustion temperature. The mass flow rates presented in Fig. 8 can be extended to include the foregoing range of combustion temperatures and gas constants by applying the reciprocal of the aforementioned correction factor, predicting the actual mass flow rate within ± 0.25%.

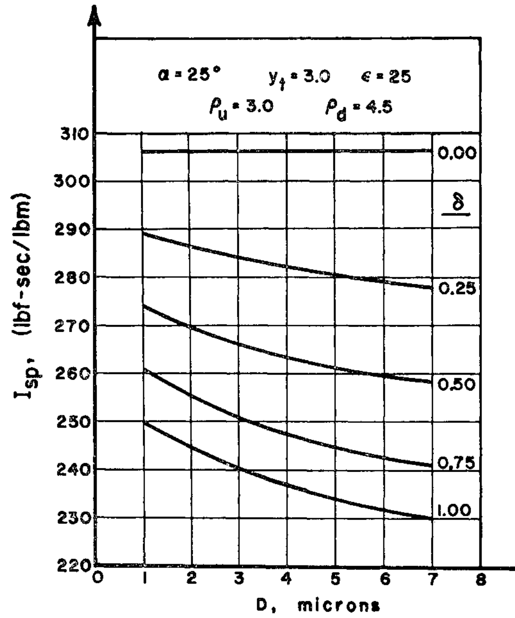

Fig. 3 Specific impulse vs particle diameter for 25° nozzle.

The combustion pressure is 500 psia in all of the figures presented herein. Combustion pressures in the range 250–1000 psia were investigated and found to have an insignificant effect on specific impulse. The mass flow rates presented in Fig. 8 can be utilized to predict the mass flow rates in this pressure range within ± 0.25% by applying the correction factor $(P_c/500)$ based on the ratio of combustion pressures. In all cases the ambient pressure is a vacuum. Specific impulses at other ambient pressures can be calculated by subtracting the pressure term from the values presented. Thus,

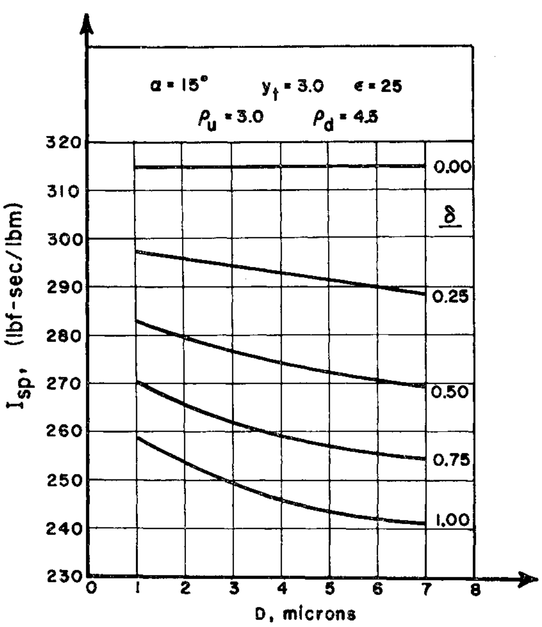

Fig. 2 Specific impulse vs particle diameter for 15° nozzle.

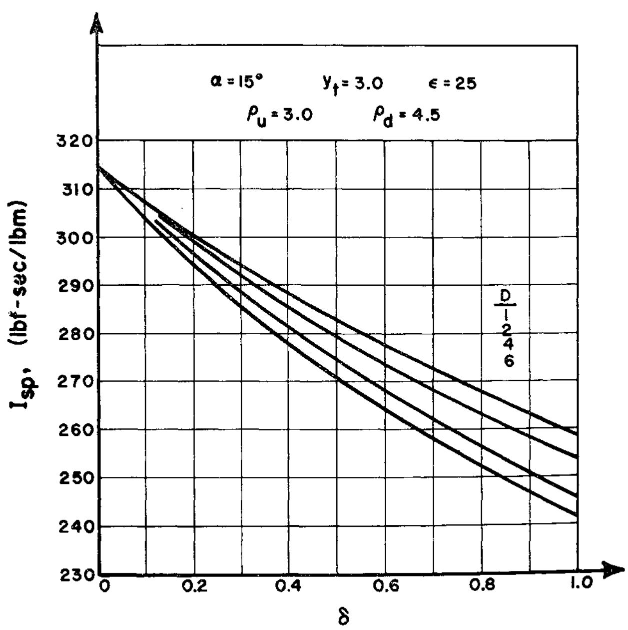

Fig. 4 Specific impulse vs flow rate ratio for 15° nozzle.

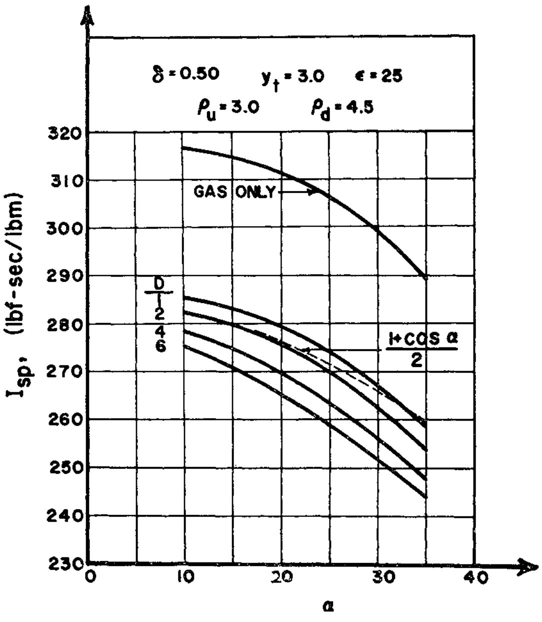

Fig. 5 Specific impulse vs cone angle.

the results presented in Figs. 2–8 can be utilized to predict the performance of conical nozzles over wide ranges of particle sizes and flow rate ratios, combustion temperatures and pressures, gas molecular weights, and ambient pressures.

The values of particle throat velocity lag presented in Fig. 9, and utilized to calculate the modified perfect gas initial value line, were determined by solving the one-dimensional gas-particle flow equations in the subsonic portion of the nozzle. All of the subsonic portions of the nozzles were geometrically similar, having an inlet semiangle of 40°, $(\rho_u/y_t) = 1.0$, and an inlet area ratio of 9 to 1. Gas-particle equilibrium was assumed to exist at the nozzle entrance. The results of Fig. 9 are for a flow rate ratio of 0.5 except as noted for $D = 2\mu$. The results for the other particle diameters at other flow rate ratios were similar to those for $D = 2\mu$.

The mass flow rate curves presented in Fig. 8 were calculated for the subsonic geometry just described. For particle diameters of 2μ, the mass flow rate was found to be proportional to the 1.99 power of the throat radius. For particle diameters of 6μ, the mass flow rate is proportional to the 1.98 power of the throat radius. Such results are consistent with gas-only nozzle flows where the mass flow rate is proportional to the square of the throat radius. As discussed previously, these flow rate results can be utilized to predict the flow rate over a wide range of typical operating conditions by the application of appropriate correction factors.

Figures 2 and 3 present specific impulse vs particle diameter with flow rate ratio as a parameter for a 15° and 25° conical nozzle, respectively. Both nozzles have $y_t = 3.0$, $\rho_d = 4.5$, and an expansion ratio of 25. A considerable performance loss with respect to the gas-only result is seen to exist when particles are present, the magnitude of the loss increasing almost linearly with flow rate ratio for a given value of particle diameter, and almost linearly with particle diameter for a given value of flow rate ratio. Figure 4 is a cross-plot of Fig. 2 which more clearly illustrates the effect of flow rate

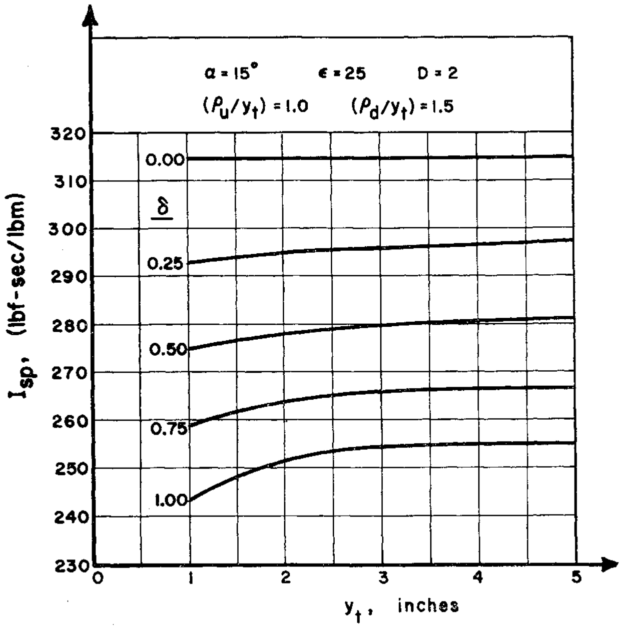

Fig. 7 Specific impulse vs throat radius for 15° nozzle.

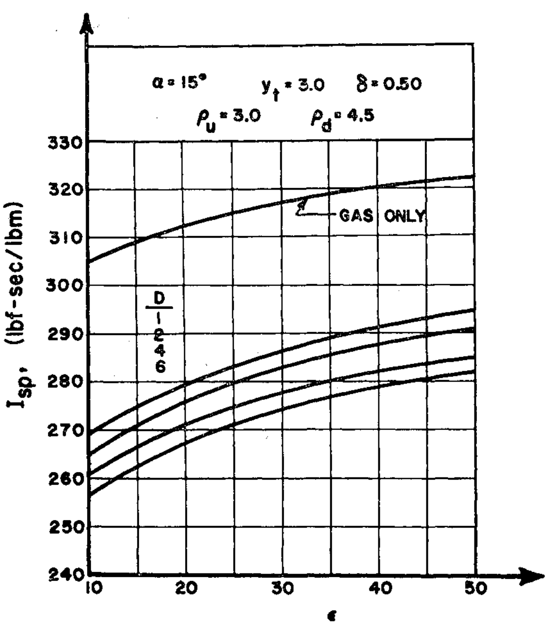

Fig. 6 Specific impulse vs expansion ratio for 15° nozzle.

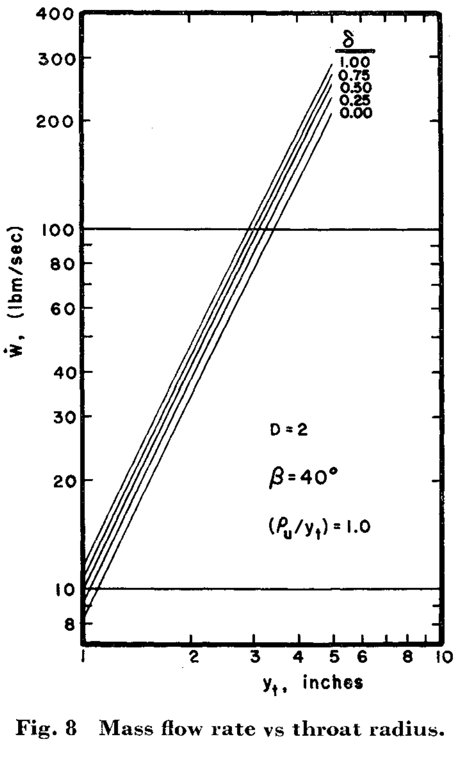

Fig. 8 Mass flow rate vs throat radius.

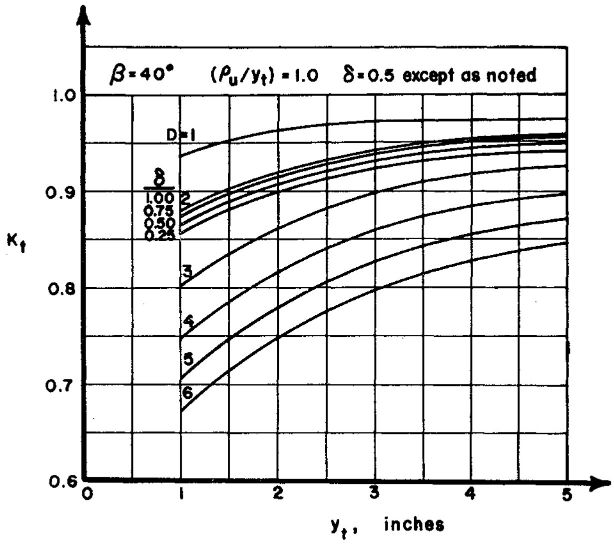

Fig. 9 Particle throat velocity lag vs throat radius.

ratio on specific impulse for a 15° conical nozzle. No comparison was made with thermochemical calculations due to the complex manner in which flow rate ratio affects chamber temperature, an effect neglected in the present study. Such an analysis, which assumes kinetic and thermal equilibrium between the phases, would give more meaningful results with which to evaluate the magnitude of particle nonequilibrium flow losses.

In Figs. 2–7, a great amount of caution must be exercised in interpreting the effect of flow rate ratio. In all of the results presented herein, specific impulse decreases as flow rate ratio increases, a trend certainly not observed in actual motors. This paradox results from assuming a fixed chamber temperature for all flow rate ratios. Actually, the chamber temperature would increase as the flow rate ratio is increased, thus increasing the specific impulse achievable with a given propellant formulation. However, in order to reduce the number of variables in this parametric study, the chamber temperature was assumed to be constant. As discussed previously, the actual mass flow rate and specific impulse at a chamber temperature other than 6000°R can be calculated by applying the appropriate correction factors. Thus the effect of a varying chamber temperature can be calculated from the results presented herein once the variation of chamber temperature with flow rate ratio has been determined from thermochemical calculations. No such results are presented in this paper.

Figure 5 illustrates the effect of nozzle semidivergence angle on specific impulse for nozzles having $y_t = 3.0$, an expansion ratio of 25, and a particle flow rate ratio of 0.5. The performance loss increases rapidly with increasing semidivergence angle, the particle size exerting a lesser but significant effect. The accuracy of Fig. 5 will decrease as the cone angle increases and the throat downstream radius of curvature decreases. In order to evaluate the effect of cone angle accurately over a wide range of throat configurations, a more sophisticated subsonic solution is required. The dotted curve on Fig. 5 is based on the conventional divergence loss factor $(1 + \cos\alpha)/2$, with the performance of the 15° nozzle and 2-μ-diam particle chosen as a reference. It is seen that the performance loss is slightly larger than that predicted by the preceding correction factor. Similar results were obtained for all particle diameters and flow rate ratios.

Figure 6 illustrates the effect of the expansion ratio of a 15° conical nozzle on specific impulse for a flow rate ratio of 0.5. As expected, increasing the expansion ratio increases the specific impulse, the amount of the increase gradually decreasing for larger expansion ratios. Particle size again exerts a lesser but significant effect on the results.

Figure 7 presents the effect of geometrical scale factor in the form of throat radius on the specific impulse of a 15° conical nozzle for $(\rho_d/y_t) = 1.5$, expansion ratio of 25, and particle diameter of 2μ. A slight increase in specific impulse is evident for larger nozzles. However, the increase is slight for throat radii greater than 3 in. The predominating factor as usual is the flow rate ratio.

Results similar to those presented in Figs. 5–7 were obtained for other particle diameters and flow rate ratios. However, the trends were identical, and so those curves are not presented. In all of the figures presented, the gas-only performance is included to illustrate the penalty due to the presence of the particles.

Discussion

The data presented in Figs. 2–7 are representative of the effects encountered in gas-particle conical nozzle flows. Different inlet and throat geometries were investigated and were found to have effects slightly different in magnitude, although the general trends were the same as those presented here. Contoured nozzles were not investigated, but it is expected that, with the exception of Fig. 5, which would no longer be applicable, effects similar in trend and magnitude to those presented would occur. No comparison with one-dimensional results has been made. It is felt that such an analysis would retain all of the qualitative features found in the present investigation. For current high-energy solid propellants, the data presented herein should provide an accurate prediction of the effect of various gasdynamic and geometrical parameters on the specific impulse delivered by the thrust nozzle.

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